Eur. Phys. J. B **47**, 109–114 (2005)
DOI: 10.1140/epjb/e2005-00309-x

THE EUROPEAN PHYSICAL JOURNAL B

Current reversal in a two-noise ratchet

Baoquan Ai[1,a], Huizhang Xie[2], and Lianggang Liu[3]

[1] School of Physics and Telecommunication Engineering, South China Normal University, 510631, GuangZhou, China
[2] Department of Physics, South China University of technology, 510641, GuangZhou, China
[3] Department of Physics, ZhongShan University, 510275 GuangZhou, China

Received 27 April 2005 / Received in final form 12 July 2005
Published online 21 September 2005 –

Abstract. Transport of a Brownian particle moving in a periodic potential is investigated in the presence of the two correlated noises. We present the analytical expression of the net current at quasi-steady state limit. The competitions among the asymmetric parameter of the potential, the noise correlation parameter and the temporal asymmetric parameter of driving force lead to the phenomena like current reversal. The competitions of different driving factors are necessary for current reversal.

PACS. 05.40.-a Fluctuation phenomena, random processes, noise, and Brownian motion – 02.50.Ey Stochastic processes – 87.10.+e General theory and mathematical aspects

Introduction

Transport phenomena play a crucial role in many processes from physical, biological to social systems. There has been an increasing interest in transport properties of nonlinear systems which can extract usable work from unbiased nonequilibrium fluctuations [1–4]. This comes from the desire of understanding molecular motors [5], nanoscale friction [6], surface smoothening [7], coupled Josephson junctions [8], optical ratchets and directed motion of laser cooled atoms [9], and mass separation and trapping schemes at microscale [10].

The focus of research has been on the noise-induced unidirectional motion over the last decade. A ratchet system is generally defined as a system that is able to transport particles in a periodic structure with nonzero macroscopic velocity in the absence of macroscopic force on average. In these systems, directed Brownian motion of particles is generated by nonequilibrium noise in the absence of any net macroscopic forces and potential gradients [4]. Typical examples are rocking ratchets [4,11], flashing ratchets [12], diffusion ratchets [13], correlation ratchets [4,14]. In all these studies, the potential is taken to be asymmetric in space. It has also been shown that a unidirectional current can also appear for spatially symmetric potentials if there exits an external random force either asymmetric [2] or spatially-dependent [3].

The current reversal is very important for separation of micro-particles [15]. It is also of interest in biology. Motions of macromolecules are probably responsible for the vesicle transport inside eukaryotic cells. A typical example is the motion of proteins along a microtubule, modelled usually by a ratchet [29]. It is well known that the two typical proteins, kinesins and dyneins, move along tubulin filaments in opposite directions.

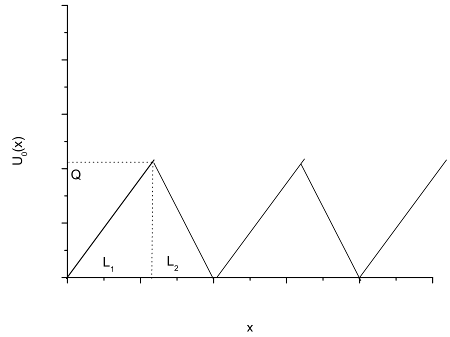
The current reversal in ratchet systems can be engendered by varying system parameters [16–28]. The current can be reversed, for example, by a noise of Gaussian force with non-white power spectrum in present of stationary periodic potential [19]. The current reversal can also be obtained in two-state ratchets if the long arm is kinked [20]. Bier and Astumian [21] have also found the current reversal in a fluctuating three-state ratchet. In the presence of a kangaroo process as the driving force, the current reversal can be triggered by varying the noise flatness, the ratio of the fourth moment to the square of the second moment [22]. The current reversal can be induced by both an additive Gaussian white and an additive Ornstein-Uhlenbeck noise in a correlation ratchet [23]. The current reversal also appear in forced inhomogeneous ratchets [17,18]. The current reversals are also frequent in the absence of noise (deterministic ratchets) [1,24].

The present work studies the current reversal of a two-noise ratchet in the presence of an asymmetric unbiased external force. When positive driving factors compete with negative ones, the current may reverse its direction. The competition between the different driving factors is necessary for the current reversal. Our emphasis is on finding conditions of generating current reversal. This is achieved by using a quasi-steady state limit to solve the Fokker-Planck equation.

Current of the two-noise ratchet

Consider a Brownian particle moving in a sawtooth potential with the correlated noises. The particle motion satisfies the dimensionless Langevin equation

[a] e-mail: aibq@21cn.com



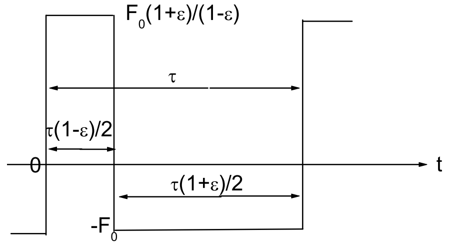


Fig. 1. Potential and Driving force: (a)potential $U_0(x) = U_0(x+L)$; $U_0(x)$ is a piecewise linear and periodic potential; the period of the potential is $L = L_1 + L_2$; $\Delta = L_1 - L_2$; Q is the height of the potential. (b) Driving force $F(t)$ which preserved the zero mean $\langle F(t)\rangle = 0$; $F(t+\tau) = F(t)$; where the temporal asymmetry is given by the parameter ε.

motion [18,30,31]

$$\frac{dx}{dt} = -\frac{\partial U_0(x)}{\partial x} + F(t) + \xi_2(t)F(t) + \xi_1(t), \quad (1)$$

where x stands for the position of Brownian particle. The geometry of potential $U_0(x) = U_0(x+L)$ is displayed in Figure 1a and $U_0(x)$ within the interval $0 \le x \le L$ is described by

$$U_0(x) = \begin{cases} \dfrac{2Q}{L+\Delta}x, & 0 < x \le (L+\Delta)/2; \\ -\dfrac{2Q}{L-\Delta}(x-L), & (L+\Delta)/2 < x \le L, \end{cases} \quad (2)$$

where L is the period of the potential, Q the barrier height of the potential, Δ the asymmetric parameter of the potential.

$F(t)$ is an external fluctuation [18] (Fig. 1b)

$$F(t+\tau) = F(t), \int_0^{\tau} F(t)dt = 0, \quad (3)$$

$$F(t) = \begin{cases} \frac{1+\varepsilon}{1-\varepsilon}F_0, & n\tau \le t < n\tau + \frac{1}{2}\tau(1-\varepsilon); \\ -F_0, & n\tau + \frac{1}{2}\tau(1-\varepsilon) < t \le (n+1)\tau, \end{cases} \quad (4)$$

where τ is its period, ε the temporal asymmetric parameter of the driving force.

$\xi_1(t)$, $\xi_2(t)$ are white noises with zero mean. They are usually treated as independent random variables in most of previous investigations. However, here we assume that the two noises are correlated with each other and the correlations between the two noises have the following form [18,31]

$$\langle \xi_i(t)\xi_j(t')\rangle = 2C_{i,j}k_B\sqrt{T_iT_j}\delta(t-t'), \qquad i = 1,2; j = 1,2, \quad (5)$$

where $C_{i,j} = \lambda$ for $i \ne j$ and $C_{i,j} = 1$ for $i = j$, λ denotes the correlation parameter between $\xi_1(t)$ and $\xi_2(t)$, and $-1 \le \lambda \le 1$. k_B is Boltzman constant and equal to 1 for simplicity, T_1, T_2 absolute temperatures.

The probability density satisfies the associated Fokker-Planck equation [30]

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}[U'(x,t) + G(F(t),\lambda)\frac{\partial}{\partial x}]P(x,t) = -\frac{\partial j(x,t)}{\partial x}, \quad (6)$$

where the prime stands for the derivative with respect to the space variable x. The probability current density $j(x,t)$ is given by

$$j(x,t) = -U'(x,t)P(x,t) - G(F(t),\lambda)\frac{dP(x,t)}{dx}, \quad (7)$$

$$U(x,t) = U_0(x) - F(t)x, \quad (8)$$

$$G(F(t),\lambda) = T_2F(t)^2 + 2\lambda F(t)\sqrt{T_1T_2} + T_1, \quad (9)$$

$P(x,t)$ is the probability density for the particle at position x and at time t. It satisfies the normalization condition and the periodicity condition,

$$P(x,t) = P(x+L,t), \quad (10)$$

$$\int_0^L P(x,t)dx = 1. \quad (11)$$

If $F(t)$ changes very slowly with respect to t, namely, its period is longer than any other time scale of the system, there exists a quasi-steady state. In this case, by following the method in [18,30,31], we can obtain the current $j(F(t))$ from equations (7–11),

$$j(F(t)) = \frac{G(F(t),\lambda)\{1 - \exp[\frac{-LF(t)}{G(F(t),\lambda)}]\}}{\int_0^L e^{\phi(x)}dx \int_x^{x+L} e^{-\phi(x)}dy}, \quad (12)$$

where $\phi(x,t)$ is generalized potential

$$\phi(x,t) = -\frac{U(x,t)}{G(F(t),\lambda)}. \quad (13)$$

After substituting $U_0(x)$ and $G(F(t),\lambda)$, we have

$$j(F(t)) = \frac{P_2^2 \sinh[LF(t)/2G(F(t),\lambda)]}{G(F(t),\lambda)(L/Q)^2P_3 - (L/Q)P_1P_2\sinh[LF(t)/2G(F(t),\lambda)]}, \quad (14)$$

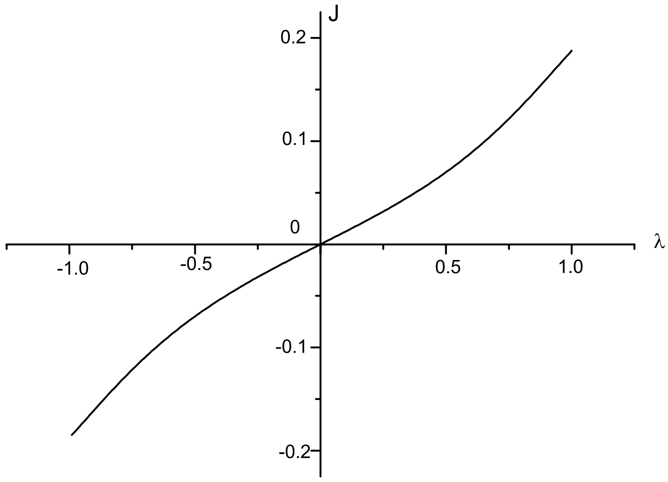


Fig. 2. Current J versus correlation parameter λ between the two thermal noises at $Q = 1$, $L = 1$, $F_0 = 0.5$, $T_1 = 0.5$, $T_2 = 0.5$, $\varepsilon = 0$ and $\Delta = 0$.

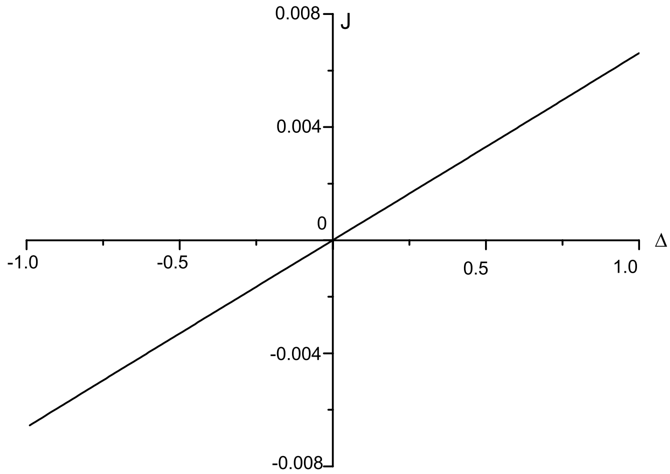


Fig. 3. Current J versus asymmetric parameter Δ of the potential at $Q = 1$, $L = 1$, $F_0 = 0.5$, $T_1 = 0.5$, $T_2 = 0$, $\lambda = 0$ and $\varepsilon = 0$.

where

$$P_1 = \Delta + \frac{(L^2 - \Delta^2)F(t)}{4Q}, \tag{15}$$

$$P_2 = \left[1 - \frac{F(t)\Delta}{2Q}\right]^2 - \left[\frac{LF(t)}{2Q}\right]^2, \tag{16}$$

$$P_3 = \cosh\left[\frac{Q - F(t)\Delta/2}{G(F(t),\lambda)}\right] - \cosh\left[\frac{LF(t)}{2G(F,\lambda)}\right]. \tag{17}$$

The average current is

$$J = \frac{1}{\tau}\int_0^\tau j(F(t))dt. \tag{18}$$

For the external force $F(t)$ shown in Figure 1b, we can have

$$J = \frac{1}{2}\left[(1-\varepsilon)j\left(\frac{(1+\varepsilon)F_0}{1-\varepsilon}\right) + (1+\varepsilon)j(-F_0)\right]. \tag{19}$$

Results and discussion

Figure 2 shows the current J as a function of the noise correlation parameter λ at $\Delta = 0$ and $\varepsilon = 0$. The current is negative for $\lambda < 0$, zero at $\lambda = 0$ and positive for $\lambda > 0$. Therefore, we can have the current reversal by changing the sign of λ, the noise correlation parameter.

Figure 3 shows the current J versus the asymmetric parameter Δ of the potential at $\lambda = 0$ and $\varepsilon = 0$. Similarly, the current is negative for $\Delta > 0$, zero at $\Delta = 0$ and positive for $\Delta > 0$. Therefore, the current can reverse its direction by changing the sign of Δ.

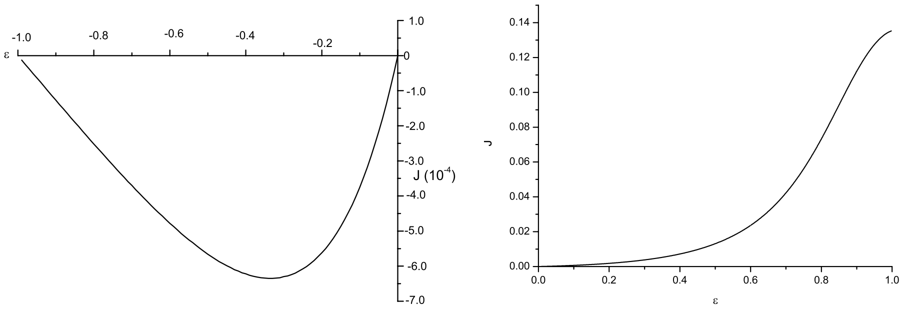
Figure 4a show the current J as a function of the negative temporal asymmetric parameter ε of the driving force at $\lambda = 0$ and $\Delta = 0$. When $\varepsilon = -1.0$, namely, no external force acting on the system, no current occurs. When $\varepsilon = 0$, there is no any driving factors at $\lambda = 0$ and $\Delta = 0$, therefore, no current occurs, also. When $-1.0 < \varepsilon < 0$, the current is negative and has a minimum value. The current J versus the positive temporal asymmetric parameter ε is shown in Figure 4b. The current J is positive for $\varepsilon > 0$ and increases with ε. When the sign of ε is changed, the current reversal can occur. For the noise correlation parameter λ and the asymmetric parameter of the potential, the temporal asymmetric parameter ε is another way of inducing a net current.

The current J as a function of T_1 is shown in Figure 5 for different combinations of ε and Δ at $T_2 = 0$ and $\lambda = 0$. When $\lambda = 0$ and $T_2 = 0$, the correlation ratchet reduces to a rocking ratchet. When $T_1 \to 0$, J tends to zero for all values of ε and Δ. Therefore, the particle can not pass the barrier and there is no current. When $T_1 \to +\infty$ so that the thermal noise is very large, the ratchet effect disappears and $J \to 0$, also. There exists some values of T_1 at which the current J takes its maximum or minimum value. When ε is negative ($\varepsilon = -0.4$), the current may reverse its direction as increasing T_1. Therefore, there exists current reversal for $\varepsilon\Delta < 0$ at $\lambda = 0$. However, $\varepsilon\Delta < 0$ is not sufficient for current reversal. For example, the current is always negative for $\varepsilon = -0.4$, $\Delta = 0.2$ and negative for $\varepsilon = -0.4$, $\Delta = 0.3$.

In Figure 6, we plot the current J as a function of temperature T_1 for different combinations of ε and λ at $\Delta = 0$ (symmetric potential). When $T_1 \to 0$, because of existing of the other thermal noise T_2, the current tends to a negative value, instead of zero, at $\varepsilon = -0.4$, which is different from Figure 5. When ε is negative ($\varepsilon = -0.4$), the current reversal occurs for positive λ (0.4, 0.35, 0.3). Hence, the current may reverse its direction for $\varepsilon\lambda < 0$.

Figure 7 shows the current J versus the temperature T_1 for different combinations of Δ and λ at $\varepsilon = 0$. When $T_1 \to 0$, the current tends to a positive value for $\Delta = 0.8$. There exits current reversal for positive Δ and negative λ. The current may reverse its direction for $\lambda\Delta < 0$.

Figure 8 shows the current as a function of T_1 for different combinations of Δ, λ and ε. From the figure, when



(a) (b)

Fig. 4. Current J versus temporal asymmetric parameter ε of the driving force at $Q = 1$, $L = 1$, $F_0 = 0.5$, $T_1 = 0.5$, $T_2 = 0$, $\lambda = 0$ and $\Delta = 0$: (a) ε is negative. (b) ε is positive.

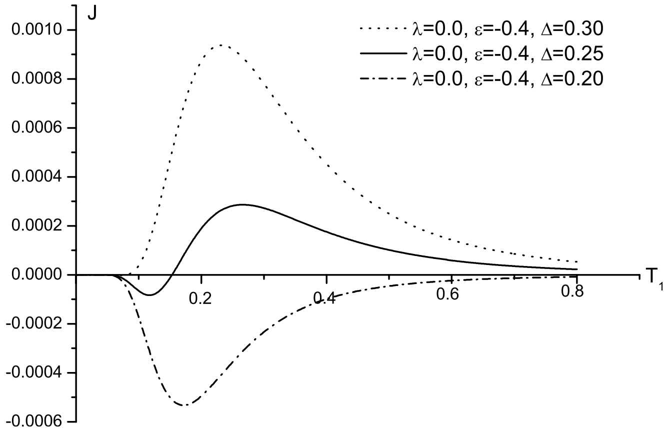


Fig. 5. Current J versus temperature T_1 for different combinations of ε and Δ at $Q = 1$, $L = 1$, $F_0 = 0.5$, $T_2 = 0$ and $\lambda = 0$.

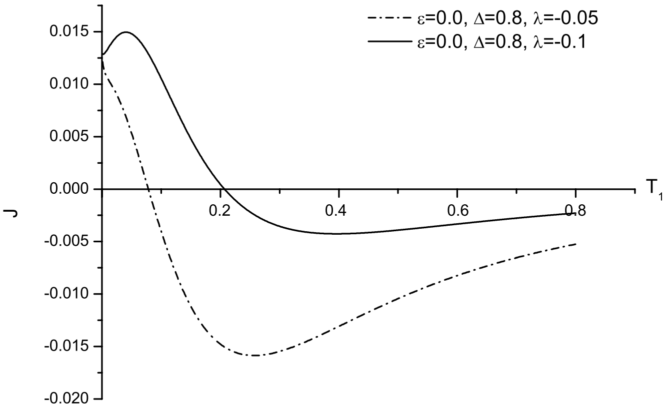


Fig. 7. Current J versus temperature T_1 for different combinations of Δ and λ at $Q = 1$, $L = 1$, $F_0 = 0.5$, $T_2 = 0.5$ and $\varepsilon = 0$.

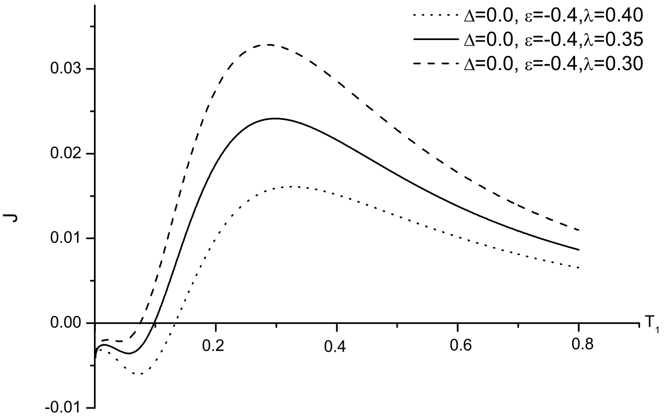


Fig. 6. Current J versus temperature T_1 for different combinations of ε and λ at $Q = 1$, $L = 1$, $F_0 = 0.5$, $T_2 = 0.5$ and $\Delta = 0$.

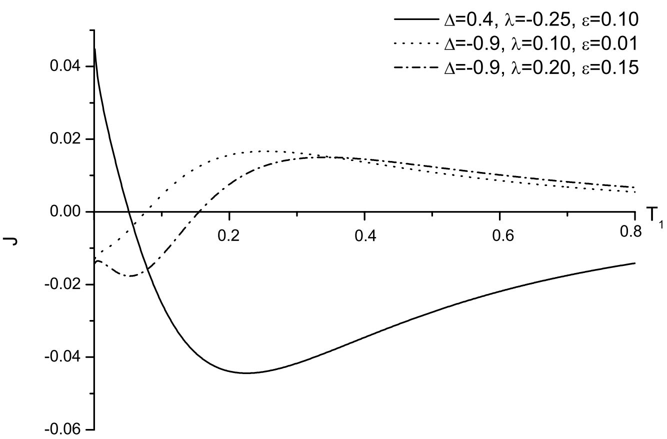


Fig. 8. Current J versus temperature T_1 for different combinations of ε, Δ and λ at $Q = 1$, $L = 1$, $F_0 = 0.5$ and $T_2 = 0.5$.

a negative driving factor meets the two positive driving factors, or a positive driving factor meets the two negative driving factors, the current reversal may occur. Hence, the current may reverse its direction as increasing temperature T_1 for $\Delta\varepsilon\lambda < 0$.

From Figure 2 to Figure 8, when the system controlled by two driving factors (Figs. 5–7), the current may reverse its direction for the two competitive driving factors ($\Delta\varepsilon < 0$, $\Delta\lambda < 0$, $\lambda\varepsilon < 0$). When the system is under three driving factors (Fig. 8), the current reversal may occur for at least two opposite driving factors.

Concluding remarks

We study the transport of a Brownian particle moving in a periodic potential in the presence of an asymmetric unbiased external force and two correlated noises. We obtained the current analytically in a quasi-steady state limit. It is found that the asymmetric parameter Δ of the potential, the noise correlation parameter λ and the temporal asymmetric parameter ε of the external unbiased force are the three pivotal factors for obtaining a net current. For the three positive or the three negative driving factors, current cannot reverse its direction. The current reversal cannot occur either if there is only one driving factor. Two or three opposite driving factors are necessary for current reversal.

Our ratchet model is proposed in an attempt of describing molecular motor in biology systems. The potential describes the track and the structure of the motor. The external force depicts the stroke force due to ATP hydrolyzing. Usually, due to the fluctuation of the condition, there are two noises in the system. One is the fluctuation of the stroke force, multiplicative noise. The other is the inherent fluctuation of the system, additive noise. Because the two noises may have the common origin, we assume that they are correlated with each other. For such a ratchet system, there exist three factors for obtaining a net current: space asymmetry in the potential, temporal asymmetry in the external force and noise correlation. For a symmetric external force and single noise ratchet, asymmetry in potential is sufficient for a net transport [11]. For a symmetric potential and single noise ratchet, asymmetry in the external force can induce a net current [18]. Noise correlation can break the symmetry of the generalized potential and induce a net transport even if the potential and the external force are symmetric [31]. When these factors meet, the competitions between them may induce current reversals.

The current reversal is very important in new particle separation devices such as electrophoretic separation of micro-particles [15]. To date, the feasibility of particle transport by man-made devices has been experimentally demonstrated for several ratchet types. Beyond the separation methods, the phenomena of current reversals may be of interest in biology [32], e.g., when considering the motion of macro- molecules. Myosin moves along actin filaments towards their plus extremity, and kinesins and dyneins move along tubulin filaments towards their plus and minus extremities respectively. It is well known that the two current reversals effect allows one pair of motor proteins to move simultaneously in opposite directions along the microtubule inside the eukaryotic cells. Several biological molecular motors, for instance kinesin and nonclaret disjunctional, belonging to the same superfamily of motor proteins move towards opposite ends of the microtubules. The ratchet mechanism was used for obtaining efficient separation methods of nanoscale objects, e.g., DNA molecules, proteins, viruses, cells, etc. These can be explained by the current reversal.

To summarize, it is remarkable that the interplay among the three different driving factors generates a rich variety of cooperation effects such as current reversal. We expect that our analysis should be applicable for particle separation devices, control of molecular motors and other microscale phenomena.

References

1. J.L. Mateos, Phys. Rev. Lett. **84**, 258 (2000)
2. F. Marchesoni, Phys. Lett. A **119**, 221 (1986)
3. M. Borromeo, F. Marchesoni, Phys. Lett. A **249**, 199 (1998)
4. P. Hanggi, F. Marchesoni, F. Nori, Ann. Phys. (Leipzip), 1–20 (2005)
5. J. Maddox, Nature **368**, 287 (1994); R.D. Astumian, I. Derenyi, Eur. Biophys. J. **27**, 474 (1998); N. Thomas, R.A. Thornhill, J. Phys. D **31**, 253 (1998); C.R. Doering, B. Ermentrout, G. Oster, Biophys. J. **69**, 2256 (1995)
6. J. Krim, D.H. Solina, R. Chiarello, Phys. Rev. Lett. **66**, 181 (1991); J.B. Sokolo, J. Krim, A. Widom, Phys. Rev. B **48**, 9134 (1993); L. Daikhin, M. Urbakh, Phys. Rev. E **49**, 1424 (1994); C. Daly, J. Krim, Phys. Rev. Lett. **76**, 803 (1996); M.R. Sorensen, K.W. Jacobsen, P. Stoltze, Phys. Rev. B **53**, 2101 (1996)
7. I. Derenyi, C. Lee, A.-L. Barabasi, Phys. Rev. Lett. **80**, 1473 (1998)
8. I. Zapata, R. Bartussek, E. Sols, P. Hanggi, Phys. Rev. Lett. **77**, 2292 (1996)
9. L.P. Faucheux, L.S. Bourdieu, P.D. Kaplan, A.J. Libchaber, Phys. Rev. Lett. **74**, 1504 (1995); C. Mennerat-Robilliard, D. Lucas, S. Guibal, J. Tabosa, C. Jurczak, J.-Y. Courtois, G. Grynberg, Phys. Rev. Lett. **82**, 851(1999)
10. L. Gorre-Talini, J.P. Spatz, P. Silberzan, Chaos **8**, 650 (1998); I. Derenyi, R.D. Astumian, Phys. Rev. E **58**, 7781 (1998); D. Ertas, Phys. Rev. Lett. **80**, 1548 (1998); T.A.J. Duke, R.H. Austin, Phys. Rev. Lett. **80**, 1552 (1998)
11. M.O. Magnasco, Phys. Rev. Lett. **71**, 1477 (1993)
12. P. Hanggi, R. Bartussek, Lecture Notes in Phys. **476**, 294 (1996)
13. P. Reimann, R. Bartussek, R. Haussler, P. Hanggi, Phys. Lett. A **215**, 26 (1994)
14. C.R. Doering, W. Horsthemke, J. Riordan, Phys. Rev. Lett. **72**, 2984 (1994)
15. C. Kettner, Phys. Rev. E **61**, 312 (2000)
16. R. Tammelo, R. Mankin, D. Martila, Phys. Rev. E **66**, 051101 (2002)
17. D. Dan, M.C. Mahato, A.M. Jayannavar, Phys. Rev. E **63**, 056307 (2001)
18. B.Q. Ai, X.J. Wang, G.T. Liu, H.Z. Xie, D.H. Wen, W. Chen, L.G. Liu., Eur. Phys. J. B **37**, 523 (2004); B.Q. Ai, G.T. Liu, H.Z. Xie, L.G. Liu, Chaos **14**, 957 (2004); B.Q. Ai, X.J. Wang, G.T. Liu, L.G. Liu, Phys. Rev. E **68**, 061105 (2003); B.Q. Ai, X.J. Wang, G.T. Liu, L.G. Liu, Phys. Rev. E **67**, 022903 (2003); D.R. Chialvo, M.M. Millonas, Phys. Lett. A **209**, 26 (1994)
19. M.M. Millonas, M.I. Dykman, Phys. Lett. A **185**, 65 (1994)
20. J.-F. Chauwin, A. Ajdari, J. Prost, Europhys. Lett. **32**, 373 (1995)
21. M. Bier, R.D. Astumian, Phys. Rev. Lett. **76**, 4277 (1996)
22. C.R. Doering, W. Horsthemke, J. Riordan, Phys. Rev. Lett. **72**, 2984 (1994)

23. R. Bartussek, P. Reimann, P. Hanggi, Phys. Rev. Lett. **76**, 1166 (1996)
24. M. Borromeo, G. Costantini, F. Marchesoni, Phys. Rev. E **65**, 041110 (2002)
25. R. Mankin, A. Ainsaar, A. Haljas, E. Reiter, Phys. Rev. E **63**, 041110 (2001)
26. J. Kula, T. Czernik, J. Luczka, Phys. Rev. Lett. **80**, 1377 (1998); M. Kostur, J. Luczka, Phys. Rev. E **63**, 021101 (2001)
27. I. Derenyi, A. Ajdari, Phys. Rev. E **54**, R5 (1996)
28. F. Marchesoni, Phys. Lett. A **237**, 126 (1998); H.A. Larrondoa, Fereydoon Family, C.M. Arizmendia, Physica A **303**, 67 (2002)
29. G. Oster, H. Wang, *How protein motors convert chemical energy into mechanical work.* In *Molecular Motors*, edited by M. Schliwa, ISBN 3-527-30594-7 (Wiley-VCH, 2002), pp. 207–228
30. H. Risken, *The Fokker-Planck Equation* (Springer-Verlag, Berlin, 1984)
31. Y. Jia, J.R. Li, Int. J. Mod. Phys. B **14** (5), 507 (2000); Y. Jia, L. Cao, J.R. Li, Phys. Rev. A **51**, 3196 (1995); Y. Jia, J.R. Li, Phys. Rev. Lett. **78**, 994 (1997)
32. U. Henningsen, M. Schliwa, Nature **389**, 93 (1997)